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# **1. Literature Review on Credit Scoring**

## **Literature background**

### **1.1.1 History of Credit Scoring**

Credit has been present in human history since the beginning of the civilisation, it can be dated back to 5000 years ago (Thomas et al., 2017). During the ancient Mediterranean period, the economy was fully based on agriculture, and it was the main source of wealth. Agricultural economies were characterised by having many ups and down due to seasonal changes, and this affected buyers' timing of payments. Some people had surplus of crops, while others had the need for this surplus but not the means to pay for it. Earliest references to banking and credit arrangements are exposed on stone tables in 2000 B.C (Lewis, 1992) as well as in the Code of Hammurabi around 1726 B.C, and it is fully extended until the end of the roman empire in which money was created, and credit started to be known as today (MacDonald & Gastmann, 2017).

Creditworthiness scoring has been among the oldest methods both in data analytics and risk management, and it is essential towards evaluating credit applications (Thomas et al., 2017). Credit scoring as we know it today dates to the 1950s. Back then there was no such thing as credit score. For people to get loans they had to hold long interviews with credit officers at the banks in which a decision was made based on individual judgement. Regarding this old system there were 2 big problems, first, the loan decision depended on a subjective judgement which was not an accurate way of determining whether a borrower will pay back a loan or not, second, the discriminatory bias regarding racial and gender characteristics of the applicants (Abdou & Pointon, 2011).

Although credit scoring as of today is quite new, the literature points out the existence of credit scoring attempts in the USA in the late 19th century during the creation of the first credit bureau in the east coast. They collected information about people to sell to landlords, retailers, anyone who can be interested. Most of the information collected was related to consumption habits, debts, gambling and drinking issues of the person in question (Lauer, 2017). However, the information was not statistically speaking analysed during the decision-making process, but more in a subjective way. Later, through the development of technology and the ability of companies and banks of gathering data some first quantitative approaches arose to credit scoring in the 1930s. Some retailers, banks and financial institutions introduced a point-based system for identifying who would and would not pay back a credit based on an application form that included information on the applicant’s occupation, age, race, marital status, income, neighbourhood of residence, etc. (Lauer, 2017). Also, the first records of statistical analysis on credit data were done by David Durant which used the discriminant analysis techniques to analyse instalment loan data and determine whether some loans where good or bad (Anderson, 2007).

In the 1950s Bill Fair and Earl Isaac created the Fair, Isaac and Company (FICO) with the aim of creating an impartial and standardised scoring system. During the launch of the credit scoring system only the American Investment Company decided to use the scoring method that was based on statistical analysis. There was however a huge resistance to use such system by banks since there were not the means to apply it out of the papers. However, it happened after the 1960s when companies started to computerise data from the customers and the FICO scoring system widespread throughout the USA in which some standardisations of defaults were brought into action, for example, the ones to mark late payments as 30, 60 or 90 days behind. By this time most of credits were granted to companies and the share of granted credits to individuals was too small.

During the 1960s and the arrival of credit cards, banks realised the importance of credit scoring, the number of new applicants for credit cards made it impossible for banks to have the credit scoring as a non-automatised lending decision. The growth of computing power made popular the credit scoring adoption by the 1980s and used the system not only for credit cards but for other financial products for example, mortgages, personal loans, etc. Also, by this time new statistical techniques were introduced, more specifically, the logistic regression and linear programming. Those techniques are still being used, however, as computers and technology develops, newer and more precise techniques are put into action nowadays that provide higher quality results in terms of credit scoring, for example, artificial intelligence (AI) and machine learning (ML).

### **1.1.2 What is Credit Scoring**

Credit scoring can be defined as a set of statistical techniques in a form of a set of decision models that aims to determine whether a lender will grant a loan to a borrower. These techniques are used to establish how much of the loan credit a borrower should get and magnify the profitability of the lenders (Thomas et al., 2017). Credit scoring helps to assess the level of creditworthiness of the borrower by determining the probability of default to a certain loan.

Being creditworthy is not an attribute of a certain person, however, it is an assessment done by the lender when evaluating the profile of the borrower by the usage of credit scoring techniques. In other words, credit scoring determines the probability that a borrower will be “good” or “bad” (creditworthy or uncreditworthy) based on the profile of the borrower, as well as the economic scenario, potential loses, churns, and approval rates. All these factors together are relevant in the credit adjudication process (Siddiqi, 2013).

Businesses’ mission is to create value and maximise profits (Handy, 2002). Identify those “good customers” is very important in the financial industry, to this, banks and risk managers constantly use credit scoring techniques to select those “right” customers (low-risk customers) and implement a marketing strategy to offer financial products and get some profit. It is important to remark that the main source of revenue of banks is lending money and charging an interest rate to the borrower based on the probability of default of the loan (risk), this interest rate in known as the price of money, therefore the higher the risk of the loan the higher the interest rate related to it (Lee & Hogarth, 2018). For banks to maximise profits they need to lend as much as possible to low-risk customers, or to reduce cost related to the lending process by limiting granting loans to customers with a high probability of default (high-risk). The problem is that sometimes these low-risk customers do not need loans or financial products and they reject the banks offers regardless the effort put in the marketing campaign (Siddiqi, 2017), and that’s why banks and financial institutions make a huge effort in identifying those high-risk customers that may default a loan.

Banks and financial institutions have a huge history of data of their customers, along with the application forms when applying for a loan. The information gathered by the banks is presented generally as a form of a scorecard where the characteristics of the profile of a given customer (borrower) have a score, and the sum of these scores determines whether a loan will be granted or not to a customer. Generating a scorecard is very important process for a bank and for this many statistical and data mining techniques are used (Thomas et al., 2017). Ultimately, credit scoring is used to facilitate decision-making in business, although it is often associated with the statistical techniques and processes used in the scorecard’s development.

### **1.1.3 What are Scorecards**

As explained before, credit scoring uses predictive statistical methods to classify borrowers by their probability of being a “good” or “bad” in the future, based on the lender’s past experiences and the profile of the borrower. Credit scoring models are presented in different forms, but the most are presented in form of a regression (Anderson, 2007).

|  |  |
| --- | --- |
|  | (1) |

Where is the dependent variable associated with the outcome that refers to the probability of default, being 0 for “bad” and 1 for “good” customer, it may also come as a form of “logit” or “probit” depending on the model specifications. Being is the set of independent variables as a form of an original, transformed, or dummy variable. stands for the regression coefficient by which the independent variables are weighted indicating the relative importance and is the regression error which cannot be captured by the model.

Regression coefficients are used to present a model that attempts to explain the relationship between the independent variables and the independent variable **.** Traditional scorecards use classed variables in which scores are given if certain condition holds true based on the model coefficients, for example:

|  |  |  |
| --- | --- | --- |
| Condition | Action | Score |
| Age < 25 | Deduct | 20 |
| Age > 40 | Add | 25 |
| If owning a house | Add | 40 |
| If renting a house | Deduct | 15 |
| If married | Add | 20 |
| If single | Deduct | 20 |
| etc | … | … |

Source: own work

The scorecards can be also presented in a tabular format as shown in the next table in which the characteristics are compressed, and attributes are columns assessed on score (points) inspired on a FICO scorecard:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Characteristic | Attributes | | | | | Points |
| Years at address | <3 years  30 | 3 – 6 years  36 | >6 years  38 |  | Blank  20 | 30 |
| Years at  employer | <2 years  30 | 2-8 years  39 | 9-20 years  43 | >20 years  69 | Blank | 43 |
| Accommodation status | Own  41 | Rent  32 | Parents  35 |  | Other  36 | 32 |
| Civil status | Married  40 | Single  22 | Divorced  32 |  | Blank  32 | 32 |
| Past experience | None  3 | New  15 | Updated  30 | Past due  -5 | Write-off  Reject | 15 |
|  |  |  |  |  | Final Score | 152 |

Source: based on Anderson, 2007.

The scorecard characteristics are usually obtained from different sources of data to which the lender has access during the application process. For building a scorecard, it is considered behavioural, financial, socio-economic, and demographic data (Martin & Evzen, 2006). Regarding the sources of this data, most of the data used for evaluating the application comes internally, from the application form, credit bureaus, public commercial registers, financial statements, tax statements, integrators of economic information, and national statistical offices (Kaszyński et al., 2021). Naturally, the variables to consider may vary if the loan applicant is an individual or a company. To this, Kaszyński et al. (2021) defines a list of potential variables to include in the traditional scorecard depending on the type of applicant. For individuals, it is important to know the main source of income, disposable income, alternative sources of income, type of activity, stability of fixed expenses, assets, saving rate, credit history, delinquencies, and geolocation. However, for companies a lender would be also interested in knowing the nature of business, business history, management board, balance sheet, income statement, cashflow, suppliers, clients, delinquencies, and exposure to cross border risks.

Based on the score that a borrower gets from the application form a lender may decide to grant a loan or reject it. In particular, the total score is a measure of the risk associated with the loan for an application, and the lender (depending on if is risk adverse or risk lover) may grant high-risk loans that comply with a certain score cut off to which higher interest rate will be charged, or simply reject it.

For example, based on the previous table, the lender may decide to reject all loans that score less than 150 points, those between 150 – 160 points may be charged a higher interest rate, and those will a score higher than 160 will be granted a loan automatically. For those high-risk applicants, the lender doesn’t necessarily need to charge a higher interest rate, but may request extra conditions such as assigning a lower loan/credit, charge a higher premium on insurance, ask for a default insurance, request for extra documentation on assets, etc. The “due diligence” policy may depend on the expected approval rate and revenue or profit potential at each risk level (Siddiqi, 2017).

The attributes in the scorecard are assigned points based on statistical techniques. There are different ways to calculate the weights of the attributes in a scorecard depending on the kind of credit scoring model used, and these models provide the predictive strength of the characteristics. To this, the literature defines 2 kind of predictive modelling techniques in credit scoring: parametric modelling that makes assumptions on the data, and non-parametric modelling which doesn’t make any assumptions on the data. (Abdou & Pointon, 2011; Anderson, 2007; Lauer, 2017). Also, there are other factors considered when calculating the scores of the attribute, such as the correlation between them, and operational factors (Siddiqi, 2013). Lastly, the total score is calculated by summing all scores of the attributes of the applicant.

### **1.1.4 Credit scoring modelling characteristics**

Different statistical methods have arisen over the years, at the very beginning it was mentioned that basic statistical tools were used for credit scoring, and more sophisticated statistical methods were taking over. Actually, most of the techniques used in credit scoring techniques are within the field of predictive modelling and these can be classified in 2 sections:

**Parametric modelling techniques**: The main feature about this type of modelling techniques is that they make assumptions about the data such as Linear Discriminant Analysis and Logistic Regression. The main benefit of these models is that they are simple and easy to interpret and understand, they are also fast computationally speaking, and require less data for training purposes. Although these assumptions can help us to get more interpretable estimates, they also limit what can be learned. These models are highly constrained, have a limited complexity, and it is important to mention that the literature points out that these methods many times fail to match very complex underlying processes (Puertas et al., 2003)

**Non-parametric modelling techniques**: These models don’t make assumptions on the underlying process and have more flexibility in terms of adapting any functional form from the training data. Some these models are Pondered Regression, Regression Trees, Algorithm C4.5, Multivariate Adaptative Regression Splines, Random Forest, Extreme Gradient Boosting, Support Vector Machines, and Neural Artificial Nets (Kennedy, 2013; Knutson, 2020; Puertas et al., 2003)

## **Best practices in Credit Scoring**

Credit scoring heavily depends on data and during the scorecard development there are many data considerations. With the collected data the lender can build a credit scoring model that aims to predict future default events by using statistical methods. When building a credit scoring model some factors must be taken into account so the reliability of the final model will not be compromised. In this sense, the literature highlights 4 important factors to consider (Abdou & Pointon, 2011; Anderson, 2007; Martin & Evzen, 2006; Onay & Öztürk, 2018; Siddiqi, 2017):

* Transparency: The information can be used for assessment,
* Structure: The data is easy to analyse data considering its form,
* Data quantity: There are enough observations for model development,
* Data quality: consistency and accuracy of the data.

Anderson (2007) defines the concept of best practices as “…processes, techniques, methodologies, and the use of technology, equipment, and resources that have proven record of success at providing a desired result”. In order to ensure the quality of the credit scoring model the data must be pre-processed and analysed in advanced so that we can make sure of the precision of the estimations. Some practices and factors are industry specific. In credit scoring modelling, some general best practises in credit scoring modelling are explained by Kaszyński et al., (2021) as follows:

Firstly, for building a reliable credit scoring model we need very large datasets to ensure that our population sample for training a model can converge to the real population and therefore we can get more accurate estimates (Martin & Evzen, 2006). For this it is important to understand the target population, meaning that when building a scoring model, the scorecards shouldn’t be separated based on different products (credit card, cash loans, etc) or segmentate the scorecards based on the characteristics of the type of applicant (size of company, balance sheet, etc). This can lead to a more complex architecture, and it lowers the number of observations in each of the specific models, and therefore it affects the model stability. However, the target population should be defined considering the available information avoiding the loose of observations due to segmentation. The idea is to treat certain atypical observations (clients) in the training sample of the model as “outcome exclusions” and minimise the amount of observations that may distort the model.

Secondly, we must understand the data and assess the kind of information we get for training a scoring model. The financial and banking sector in general is vastly regulated and certain rules apply for processing and modelling the data (Knutson, 2020; World Bank Group, 2019). It is important that the training model has as many relevant explanatory features that helps to correctly assess the probability of default of a customer. Although the intuition is to include as many variables as possible, we must pay extra care when handling the data and avoid violating the privacy of the customer. European authorities warrant customer’s right to explanation and acknowledge of data processing in the latest data protection regulation (*EU General Data Protection Regulation (GDPR): Regulation (EU) 2016/679*, 2016). To this, customers must be informed that their data is being handled and assessed. Although this principle applies in the European Union, some countries are applying a similar legislation, therefore we need to make sure that our training sample does not contain data which does not comply to the current legislation (Demajo et al., 2020). If some “sensitive” information is detected in the training sample that does not comply to the legislation, then it should be removed.

Thirdly, we must keep in mind the data feed process and understand the how the data is updated in the database. The training data generally contains a combination of internal data as well as data from external sources (as explained above) and this external data in some cases can be not updated. Therefore, the training dataset must be prepared in a way that these time differences are spotted and reflected. As a rule of thumb, credit scoring models must be trained with the same data feed in which they will be used in production. Furthermore, is important to keep track of the data feed an ensure the inflow of up-to-date data and avoid situations when credit bureaus stop data feed to subscribers that breach the reciprocity agreement (Anderson, 2007).

Fourthly, we must remove visible rubbish from the data set in the sense that it must be included for training purpose those variables that are strictly relevant for modelling issues and regulatory standards. A few examples of variables that are irrelevant are those with variable coverage in time, optional variables that may come from the application form, dummy variables that indicate missing data, categorical variables with many non-hierarchical categories, customer identification data, unprocessed transaction data, and data compromising the privacy of the customer. A good practice is to create summary variables for each unprocessed transaction data that contains count and frequencies.

Fifthly, there must be a tight control on the quality of the outcome variable. Some of the good practices addressed by Kaszyński et al., (2021) in this issue are the following: get a clear definition of delinquency, mind that a restructured loan is usually a bad loan, First-in-first out (FIFO) is a better definition for a bad days past due than Last-in-first out (LIFO), there are alternative indicators for bad behaviour according to the European Bank Authority guidelines for products with compromised days past due, better to use a less precise “bad” definition than a more precise one but less consistent, the outcome variable must be defined taking into account all products on the customer level, and lastly, inactive clients should be marked as outcome exclusions.

Finally, it is important to remark the importance of Basel II compliance, and International Financial Reporting Standards (IFRS) 9. This issue will be further discussed in the next chapters.

## **Classical credit scoring approaches**

Around the 1950’s most of the credit scoring decisions were based on a subjective analysis and due to the lack of technology many processes couldn’t automatised. As explained in section 1.1.1 some attempts were done to use statistical techniques to classify customers based on their previous experience, however, the usage of statistical techniques became more popular around the 1980’s. The adoption of parametric statistical models in credit scoring started at an earlier stage with the linear discriminant analysis and later the widespread adoption of the logistic regression. These parametric models will be further explained in section 2.1.

The classic credit scoring modelling approaches consider some ideal requirements that made easy the adoption in the industry in the early 1980’s. Mainly, these models were suitable for the task at hand, which is a classification problem, also these methods sometimes are fast to learn from the data and quick to develop in production. Also, these methods can accommodate problems that are specific to a particular development, and lastly, the output of these models is very transparent, making them easy to interpret and explain, meaning that there is not a black box behind the model explaining the underlying process (Anderson, 2007). The advantages can be summarised in high interpretability, fast implementation and therefore, easy auditability.

As mentioned before, classic scoring models are based on parametric statistical models that relies on assumptions on the training data. Manipulating the data to comply to these assumptions is a process that must be done manually compared to more advanced statistical learning techniques.

## **Drawbacks and obstacles of classical scoring approach**

## **1.5 Economics behind credit scoring**

# **2. Machine learning and AI in credit scoring**

As explained in previous sections, machine learning techniques are widely used in credit scoring due to the simplification of complex scoring decisions, aiming to generate real-time predictions and reduce credit risks. In this chapter, we will dig further into the most common modelling techniques used nowadays in credit scoring, as well as the main problems that may arise in this process.

## **2.1. Parametric models used in credit scoring**

Nowadays credit-scoring systems are based on statistical and operational research methodologies that allow automatization of the scoring process increasing productivity and predicting defaults in a faster way. They are among the most effective and lucrative uses of statistical theory (Langdon et al., 1992). In previous sections it was mentioned 2 different approaches used in credit scoring modelling. While parametric modelling techniques make several assumptions about the underlying data, non-parametric techniques make few (Anderson, 2007). The parametric models, including linear discriminant analysis, logistic regression, and regularisation methods (such as ridge regression, lasso regression and elastic net), are discussed below.

### **2.1.1. Linear discriminant analysis**

Linear Discriminant Analysis (LDA) is a widely used technique for dimensionality reduction and classification. LDA enables class separability by establishing a decision region between the various classes by maximising the distance between the means of the classes, and by minimising the between-class variance and the within-class variance (Mohanty et al., 2013). Sir Fisher R. A. (1936) developed the linear discriminant function, trying to identify the set of factors that best divided two groups using accessible attributes, and LDA came later as a simple generalisation of this function. In credit scoring, the two categories are those categorized as good and bad customers by the lender and the characteristics are the details in the application form and credit bureau information.

Chart, scatter chart

Description automatically generated

Source: own work

The above graph is an example of credit scoring in which the classes (good and bad customers) are separated by the dashed line defined by LDA. These data points are projected into the dashed line reducing this two-dimensional graph into a one-dimensional graph. That’s why this method can be considered as dimensionality reduction algorithm. However, when the mean of the distributions is the same (or shared) this algorithm fails to convert the n-dimensional data into a single one-dimensional graph that makes both classes separable such as:

Shape, rectangle

Description automatically generated

Source: own work

From a mathematical point of view, the representation of LDA consists of statistical properties of the data which are calculated per class, more specifically, the mean and the variance of each class. LDA assumes that the data follows a normal distribution and that each attribute has the same variance meaning that the values of the variables are centered around the mean by the same amount on average. Keeping that in mind it is advisable to standardise the data prior using LDA.

Let our target Y denote a random binary variable with values 1 for reference to the class “default” and value 0 for reference to the class “non-default”, and X a single continuous random variable normally distributed with mean around 0 and standard deviation of 1. With the assumptions for LDA satisfied, the mean value of the X variable for each class is calculated as follows:

|  |  |
| --- | --- |
|  | (2) |

Note that k stands for each of the potential classes of the dependent variable, which in this case there are 2 classes since the dependent variable is binary. The mean is calculated dividing the sum of values by the total number of values in the corresponding class (1 or 2). The variance is calculated across all classes using the following formula:

|  |  |
| --- | --- |
|  | (3) |

The variance is calculated as the squared difference of each value from the mean within the class groups while averaging that difference across all class groups. LDA uses the Bayes theorem to estimate probabilities of a value to belong to a particular class k.

|  |  |
| --- | --- |
|  | (4) |

Being the probability of a given class, denoted as the ratio of this observations with this class in the training dataset. Since the independent variable follows a normal gaussian distribution, we can estimate which is the estimated probability of the value belonging to the class k. The denominator normalizes across for each class . When inserting the Gaussian into equation (4) and simplifying we obtain the discriminate function for class k.

|  |  |
| --- | --- |
|  | (5) |

The discriminate function is calculated for each class, and the highest value of this function for a particular class will define the output classification (Brownlee, 2016).

Based on that, we can highlight that linear discriminant analysis (LDA) uses the training observations to determine the location of a boundary between the response classes. The boundary location is determined by treating the observations of each class as samples. Theoretically, we could fit an (n) dimensional normal distribution from a multidimensional normal distribution to the observations in each class. Calculating this involves calculating the mean vector and covariance matrix for each class, as these determine the centre and shape of the distribution, respectively (Johnson & Wichern, 2002; Thomas et al., 2017). Having fitted the distributions, we could draw a boundary between the classes by selecting the set of points where the probabilities are equal. Observations on one side of the boundary would be classified as one class, and observations on the other side as the other class. We can do all this theoretically, resulting in an equation for the boundary that depends on the parameters of the fitted distribution. This means that we don't need to go through the whole process to perform discriminant analysis; we need to calculate the means and covariances and apply the formula for the boundary.

### **2.1.2. Logistic regression**

Logistic regression is by far the most widely used algorithm in credit scoring, its usage is preferred due to the simplicity and ease of interpretation. Therefore, a huge section of this dissertation is dedicated to cover the logistic regression model.

#### **2.1.2.1 Origins of logistic regression**

The logistic function was developed in the nineteenth century to describe population growth and the behaviour of autocatalytic chemical processes. Now, the growth rate of a quantity *W* (*t*) over time is given by:

|  |  |
| --- | --- |
|  | (10) |

Assuming that *Ẇ* (*t*) is proportional to *W* (*t*) we get:

|  |  |
| --- | --- |
|  | (11) |

Where *β* refers to the constant rate of growth, thus leading to the exponential growth model

|  |  |
| --- | --- |
|  | (12) |

Where the initial value *W* (0) may replace *A*. However, the exponential growth model is devoid of any upper limit. This problem was approached by the Belgian astronomer and statistician Alphonse Quetelet (1795-1874) and his student Pierre-Francois Verhulst (1804-1849) through the inclusion of an additional term, representing the increasing resistance to further growth, in equation (2) as follows:

|  |  |
| --- | --- |
|  | (13) |

Experimentation with varied forms of *φ* led to the following model when *φ* is a quadratic function:

|  |  |
| --- | --- |
|  | (14) |

Where Ω refers to the upper limit or saturation level of *W*.

In the above model, is proportional to both and . Expressing as a proportion of Ω, or, which results in the following differential equation:

|  |  |
| --- | --- |
|  | (15) |

Solving the above differential equation, we get the so-called logistic function:

|  |  |
| --- | --- |
|  | (16) |

Equation (16) was named as the ***logistic*** function by Verhulst. In regression analysis, α and *β*  may be interpreted as the intercept and the regression coefficient (or the slope of the regression line), respectively. The logistic function was rediscovered in 1920 by Raymond Pearl (1879-1940), the Director of the Department of Biometry and Vital Statistics at John Hopkins University, and his deputy Lowell J. Reed (1886-1966) while studying the United States’ population increase (Cramer, 2002).

#### **2.1.2.2. The logic of logistic regression**

Discrete or qualitative rather than continuous or quantitative events are common in many social phenomena; for example, an event happens or does not happen, a person’s life can be changed in a variety of ways that involve a characteristic, event, or choice, or large social entities such as groups, organizations and nations can arise or disintegrate, become insolvent, confront, revolt, and so on. A dichotomous indicator or dummy variable is the most common way to represent discrete binary phenomena (Pampel, 2000).

On the surface, a binary qualitative dependent variable seems to be appropriate for use in multiple linear regression. The dependent variable generally assumes only two values of zero and one. However, the estimated values for regression are in the form of mean proportions or probabilities conditional on the values of the independent variables. The regression coefficients may be interpreted as the increase or reduction in the estimated probability of possessing a characteristic or experiencing an event due to a unit change in the independent variables. However, such linear regression faces the following problems:

**Probability ceilings:** Probabilities and proportions, by definition, cannot exceed one (ceiling) or fall below zero (floor). Nonetheless, the linear regression line may stretch upward into positive infinity (or extend downward towards negative infinity) as the values of the independent variables increase (or decrease). A model may provide illogical and useless predicted values of the dependent variable greater than one and less than zero based on the slope of the regression line and the observed values of the independent variables. This can observed in the following graph.

**Additivity assumption:** Typically, linear regression assumes additivity, which states that the influence of one independent variable on the dependent variable remains constant regardless of the values of the other independent variables. When the value of one independent variable reaches a level sufficient to push the probability of the dependent variable close to one (or close to zero), the impacts of other variables have little effect. Thus, the ceiling and floor impose an intrinsic, non-additive and interactive nature on the impact of all independent variables (Pampel, 2000).

**Normality and homoscedasticity assumption:** Linear regression with dichotomous dependent variable violates the normality assumption (each value of the independent variables in the population is associated with a normal distribution of error terms around the predicted value of the dependent variable) and the homoscedasticity assumption (the dispersion of the error terms for each value of the independent variables is similar) (Pampel, 2000).

The relationship between a binary dependent variable and an independent variable is inherently non-linear. A constantly changing curve, such as the S-shaped curve, represents the relationship more smoothly and adequately than a straight line. Although other non-linear functions may depict the S-shaped curve, the logistic or logit transformation has gained popularity due to its desirable characteristics and relative simplicity since it allows to model a fitting curve between 0 and 1 as shown in the following graph.

Chart, line chart

Description automatically generated

Let *Yi* be the observations of a dichotomous dependent variable representing some event and assuming the values of 1 (occurrence of the event) and 0 (non-occurrence of the event) only. Keeping that in mind, given the probability of the occurrence of the event as *,* the probability of the non-occurrence of the event will be . In his paper, Pampel (2000) explains that the odds of the probability of the occurrence of the event relative to the probability of the non-occurrence of the event is then given by:

|  |  |
| --- | --- |
|  | (17) |

Odds are generally expressed implicitly as a ratio to one or as a single number. For instance, if the probability of an event equals 0.3, the odds are (0.3 / 0.7) or 0.428, indicating that the event occurs 0.428 times for each time it does not occur, or 428 occurrences per 1000 non-occurrences. Even though both probabilities and odds have a lower bound (floor) of zero, denoting the increasing likelihood of an event with increasingly large positive numbers, odds have no upper bound (ceiling), unlike probabilities. The transformation of probabilities into odds eliminates the probabilities’ ceiling value of one. Now, the logit or logged odds is formed by taking the natural logarithm of the odds to eliminate the odds and hence probabilities’ floor value of zero, as:

|  |  |
| --- | --- |
|  | (18) |

Based on the above statements, we can present the odds and the log of odds (logit) for a probability range from 0 to 1:

|  |  |  |
| --- | --- | --- |
| Probability | Odds | Log (Odds) |
| 0 | 0 | NEG INFINITY |
| 0.1 | 0.11 | -2.2 |
| 0.2 | 0.25 | -1.39 |
| 0.3 | 0.43 | -0.85 |
| 0.4 | 0.67 | -0.41 |
| 0.5 | 1 | 0 |
| 0.6 | 1.5 | 0.41 |
| 0.7 | 2.33 | 0.85 |
| 0.8 | 4 | 1.39 |
| 0.9 | 9 | 2.2 |
| 1 | INFINITY | INFINITY |

When plotting the probabilities against the odds we realise that it reaches 1 when the probability is 0.5, after this point the odds tends to infinity. However, the range of log of odds (logit) with respect to the probabilities has a range between negative and positive infinity, also the data is squished due to the effects of the logarithms.

Diagram, histogram

Description automatically generated

Now, getting back to our initial formulation of the binary variable with values 0 and 1. From the above explanation we know that logit of the probabilities of 0 and 1 is negative and positive infinity, which is a key part of the logistic regression process. The non-linear relationship between the independent variable (*X*) and the probability of the occurrence of the dichotomous dependent variable (*Y*) conditional on the observed values of *X* (*Xi*) expressed as *P* (*Xi*) = [ *P* (*Yi* = 1 | *X* = *Xi* )],may be transformed into a linear regression function based on the logit of *P* (*Xi*) from equation (18) as:

|  |  |
| --- | --- |
|  | (19) |

Where *α* and *β* denote the intercept and the regression coefficient (or the slope of the linear regression line), which are estimated using the maximum likelihood method, and the predictions are in form of logit.

Graphically speaking, this algorithm transforms the values of the dependent variable into the logit (that is positive and negative infinity) and fits a straight line (as in linear regression) to understand where these points would fit on this line. Then the projected values on the line are extrapolated to the Y axis providing the predicted logit for these data points of the variable X, having the actual and predicted logit based on the regression line, as shown in the following graph:

Chart, line chart

Description automatically generated

In order to convert the predicted logit back to probabilities so that it can be plotted as in graph \_\_, this algorithm uses the sigmoid function from equation (16) which is the inverse of the logit, and substitutes from equation (19) to get the logit predictions in form of probabilities as follows:

|  |  |
| --- | --- |
|  | (20) |

In that manner the predicted logit can be transformed into predicted probabilities. However, it is important to remark once again that the interpretation of the coefficients is based on the change to the predicted logit, not to the probabilities.

#### **2.1.2.3. Estimation of coefficients and goodness of fit**

In the case of a binary logistic regression model, we can estimate the coefficients using different methods. The most popular in the field of credit scoring and machine learning is the Maximum Likelihood Estimation (MLE). Due to the binary nature of the dependent variable, the error term does neither have a normal distribution nor equal variances for the values of the independent variables. As a result, the estimating approach derived from the Ordinary Least Squares (OLS) criterion, which involves minimizing the sum of the squared deviations between the observed and predicted values of the dependent variables, fails to produce efficient estimates. Therefore, rather than using OLS, logistic regression depends on maximum likelihood algorithms to estimate the coefficients.

For logistic regression, the estimation of the regression coefficients begins with an expression for the likelihood of observing the pattern of occurrences (*Y* = 1) and non-occurrences (*Y* = 0) of an event or characteristic in each sample. This expression, termed the likelihood function, depends on unknown logistic regression parameters. Maximum Likelihood Estimation (MLE) finds the model parameters that provide the maximum value for the likelihood function, thereby identifying the estimates for model parameters that are most likely to give rise to the pattern of observations in the sample data. In order to estimate the parameter of the logistic regression function, first we need to write down the log-likelihood function, then we need to optimise this function to find the values of the coefficients that maximises the log-likelihood function.

Since our dependent variable is binary, we use the corresponding density function of a Bernoulli distribution that allows us to write the probability of one data point.

|  |  |
| --- | --- |
|  | (21) |

Based on the above density function we can write the likelihood function as follows:

|  |  |
| --- | --- |
|  | (22) |

From equation (22) we can combine the products and take the log of both sides of the equation to get the log-likelihood function. Due to the properties of logarithms the products are transformed into summation as follows.

|  |  |
| --- | --- |
|  | (23) |

The above equation is the cost function to optimise in order to find the set of parameters that maximises log-likelihood. This maximisation problem could be solved by calculating the derivative of the log-likelihood function and setting it to zero, however, there is not a closed form for the maximum of this function, therefore, we must use an optimisation algorithm to compute the coefficients (Pampel, 2000). Since the log-likelihood equation is usually non-linear, general-purpose optimisation algorithms, such as Gradient Ascent, Newton-Raphson and Fisher Scoring, are also used for finding the global maxima of the function and estimating the coefficients. It is important to remark that in logistic regression using local optimisation algorithms such as gradient descent, yields in optimising the function globally.

In order to optimise equation (23) with the gradient ascent algorithm, we define the partial derivative of the log-likelihood with respect to each parameter:

|  |  |
| --- | --- |
|  | (24) |

The idea of the gradient ascent algorithm is to take the direction of the gradients (partial derivates) pointing to where the function is maximised, and continuously taking small steps in that direction until the function is maximised. This is an iterative process that updates the coefficients based on the values of the previous ones as follows:

|  |  |
| --- | --- |
|  | (25) |

Where is the step size that controls how fast the algorithm converges to the maximum. The higher the value of it is more likely to diverge from the optimal solution, whereas, if is too small, then it will take more time to converge (computationally expensive). In this case is a tuning parameter that can be set by the modeler, and mathematically speaking it only makes sense for to take values between (0, 1). According to Monroe et. al. (2017) this algorithm can be simplified with the following pseudocode:

initialise: = [0, 0, ..., 0] (m elements)

repeat many times:

gradient = [0, 0, ..., 0] (m elements)

for each training example ( , ):

for j = 0 to m:

gradient[j] +=

for j = 0 to m:

[j] += \* gradient[j]

return

Once our parameters are estimated, there are various approaches to test the Goodness of Fit (GoF) of a logistic regression model. There are some tests to assess the whether the probabilities produced by the model reflect the observed data. In statistical lingo we can distinguish between the saturated model which reflects the ideal fit meaning that the probabilities estimated by the model are equal to observed probabilities for each sample, and the estimated model:

**Deviance and Pearson :** These goodness-of-fit tests are a way to assesses how different are the current model and the full model. In this approach, the data represents the fit of the ideal model possible, the saturated model having a separate parameter for each observation. It is tested whether allparameters that are in the saturated model but not in the estimated model equal zero.

|  |  |
| --- | --- |
|  | (26) |

Where are the observed counts, are the counts estimated in the model. Also, the Deviance and Pearson χ2 follow a chi-square distribution. However, the Deviance and Pearson χ2 are not recommended goodness-of-fit statistics in a model with many predictors, especially when those are continuous. Also, it is not recommended in a model with categorical predictors with considerable number of categories. When one of these tests is rejected, we can be certain that there is a statistically significant lack of fit. Otherwise, there is no lack of fit. In other words, the null hypothesis is that the model has a good fit against the alternative hypothesis that the model has a bad fit.

**Hosmer-Lemeshow**: By contrast to the previous tests, the Hosmer-Lemeshow test compares the observed and expected frequencies of events and non-events, then it compares how the model fits the data. More specifically, it helps to determine whether the predicted probabilities deviate from the observed probabilities. First it estimates the individual probabilities sorting within response and dividing the set into ten groups with a similar number of observations from 1 to 10. Lastly it uses the groups to calculate expected counts and compare with observed counts using Pearson statistic with 10−2=8 degrees of freedom

|  |  |
| --- | --- |
|  | (27) |

If the p-value is lower than the significance level, then we reject the null hypothesis that the model fits the data nicely, meaning that there is lack of fit.

#### **2.1.2.4. Interpretation of logistic regression coefficients**

The sign of *β* in equation (20) determines whether the logit is increasing or decreasing as *X* increases. The rate of ascent or descent increases as the magnitude of *β*, that is | *β* |, increases; as *β* approaches zero, the curve flattens to a horizontal straight line. When *β* equals zero, *Y* is independent of *X*. For quantitative *X* with *β* greater than zero, the curve for *P* (*X*)has the shape of the cumulative distribution function of the logistic distribution, given by:

|  |  |
| --- | --- |
|  | (28) |

Where *μ* and *τ* are the mean and standard deviation respectively of the logistic distribution. Due to the symmetrical distribution of the logistic density, *P* (*X*)approaches one at the same rate that it approaches zero (Agresti, 2002). Multiple interpretations exist for *β* in terms of logged odds, odds and probabilities and the nature of the independent variable being continuous or dummy (Pampel, 2000). We can focus on the following ways to interpret logistic regression coefficients:

**Logged odds:** In the case of continuous independent variables, the logistic regression coefficients indicate the change in the projected logged odds of the occurrence of an event for one unit change in the independent variables. In contrast, an implicit comparison of the indicator group with the reference or excluded group is made by a one-unit change in the case of dummy independent variables. Browne (1997, p. 246), for example, uses logistic regression to forecast labour force participation for 922 female heads of home between the ages of 18 and 54 in 1989. For the continuous independent variable ‘Years employed’, the logistic regression coefficient of 0.13 indicates an increase in the logged odds of the dependent variable ‘Labor Force Participation’ by 0.13 for an additional year of employment. The author also compares the ‘Labor Force Participation’ with two dummy variables, namely ‘High school dropout’ and ‘High school graduate’ to that of the reference group consisting of women with some college education. These two dummy variables have coefficients of -1.29 and -0.68, respectively, indicating that the logged odds of being in the labor force are 1.29 lower for high school dropouts than for those with some college and 0.68 lower for high school graduates than for those with some college education (Pampel, 2000).

**Odds:** Analysing the coefficients in terms of the logged odds is not very intuitive and straightforward in order draw insights due to the logarithms, however, we can express and interpret the coefficients in terms of Odds, which is the most common way. For this, let us consider the following binary logistic regression model with two independent variables, *X*1 and *X*2 :

|  |  |
| --- | --- |
|  | (29) |

Where αis the intercept term, and *β*1 and *β*2 are the regression coefficients. Exponentiating both sides we get:

|  |  |
| --- | --- |
|  | (30) |

The equation (30) determining the odds is multiplicative, where the predicted value of the dependent variable does not change when multiplied by a coefficient of one. Therefore, the effect of each independent variable on the odds can be measured by taking the antilog of the regression coefficients. The percentage change in the odds for a one-unit change in the independent variable (with regression coefficient ‘*β‘*) which is given by .

Now it presents a more meaningful interpretation of the regression coefficients. With reference to Browne's study mentioned above, the exponentiated coefficient for the continuous variable ‘Years employed’, *e*0.13 or 1.14 indicates that a one-year increase in employment multiplies the odds of the dependent variable ‘Labour Force Participation’ by 1.14 or increases the odds by a factor of 1.14 or 14%. In the case of the dummy variable ‘High school dropout’, *e*−1.29 or 0.28, indicates that a one-unit increase in the variable multiplies the odds of ‘Labour Force Participation’ by 0.28 or the odds are 0.28 times or 72% smaller than those with some college education (reference group) (Pampel, 2000).

**Probabilities:** This approach consist in calculating at any single point of **graph\_linear\_vs\_logistic\_reg** the linear slope of the tangent of the nonlinear curve. This slope is defined by partial derivative of the nonlinear equation and describes the linear change in the probability for a one-unit change in the independent variable (Pampel, 2000). Note that it just defines the change in the tangent line due to a unit change in the independent variable, but it doesn’t define the change in the logistic regression curve, this is the so-called marginal effect. Following Browne’s example, for the variable years employed we have a logistic regression coefficient of 0.13, the expected value, the probability of event for the dependent variable is 0.83 and no event 0.17. By multiplying all three values we get a value of 0.018. Therefore, 1 year increase of employment increases the probability of event by roughly 2% at the mean.

#### **2.1.2.5. Ordinal vs Multinomial Logistic Regression model**

To better describe the difference in the logistic regression procedure when the dependent variable in ordinal or multinomial, the following table summarises the ideas behind and points out the most relevant characteristics of both approaches.

| **Ordinal Logistic Regression** | **Multinomial Logistic Regression** |
| --- | --- |
| **Logit Model:**  Ordinal logistic regression is appropriate when the outcome variable is ordinal and has more than two ordered categories. Cumulative Logit Model or Proportional Odds Model is a particular type of model that considers the ordering of categories and assumes that the odds ratio is invariant to where the outcome categories are dichotomized.  Let the model include one ordinal outcome variable *Y* having 3 ordered categories (Y = 0, 1, 2) and onedichotomousindependent variable X1 (X1 = 0, 1). Then there are two ways to dichotomize the outcome: (Y ≥ 1 vs. Y < 1; Y ≥ 2 vs. Y < 2). With this categorization of *Y*, the logit model is given by two regression equations:  **Conditional Probabilities:**  **Odds Ratio (OR):**  *OR1* =  *=*  *OR2 =* | **Logit Model:**  When the outcome variable is nominal having more than two unordered categories, multinomial logistic regression is appro-priate. Let the model include one outcome variable *Y* having 3 unordered categories (Y = 0, 1, 2) and onedichotomousindependent variable *X1*(*X1*= 0, 1). Also, let the conditional probabilities be:  Two regression equations give the logit model:  **Conditional Probabilities:**  **Odds Ratio (OR):**  *OR1* = =  *OR2* = |

*Source:* Author compilation from Kleinbaum and Klein (2010), Agresti (2002).

### **2.1.3. Regularisation methods**

In statistical learning modelling, there is a trade-off between minimising the variance and bias of a model. High model bias and low model variance oversimplifies the underlying process by increasing the difference between the average prediction and the observed values, making it hard to capture the real pattern of the data, this type of models make huge generalisations resulting in a simple model that does not learn from the training data very well, this is called underfitting in machine learning lingo. On the other hand, high model variance and low bias is a very common situation where the model learns very well from the training data but doesn’t generalise enough to unseen data, therefore there is a high error rate on the test data (Briscoe & Feldman, 2011).

In credit scoring modelling the training sets have many dimensions, and the models are exposed to the multicollinearity problem (which will be explained in further sections), also to overfitting due to the large complexity of the data (Kaszyński et al., 2021). Having very less complex data to learn from may result in a underfitted model, on the contrary, a very complex training set may result in a overfitted model. In this sense, it is important to get a right balance in the complexity of the algorithm. Regularization techniques are used in machine learning to select explanatory variables and minimise the variance due to overfitting during the training process.

#### **2.1.3.1 Ridge Regression**

Ridge regression is a regularisation technique that adds some bias to the model by imposing a penalisation term to the cost function of the model. It is usually used to estimate model parameters when there is high risk of multicollinearity that as a result could lead to highly unstable parameter estimates that are hard to interpret and to rely on (Marquardt & Snee, 1975). By imposing a penalisation term our estimates there is a reduction in the variance of the model at the cost biased estimates. In the case of using the residuals sum of square RSS as our cost function for ridge regression we can observe the role of the penalisation term in the following expression:

|  |  |
| --- | --- |
|  | (31) |

In the above equation, works as a tuning parameter that weights the sum of the squared betas, thus, it controls the amount of bias imposed to the model. In this way, the idea is to tune the parameter in a way that it won’t bias too much the estimates but at the same time it is efficient enough to considerably reduce variance.

Diagram

Description automatically generated

On the top right-hand side of the above graph, we find the maximum likelihood estimator which is the point from the coordinate (0.48, 3.2), the contour curves around this point are the projection of the likelihood function. The aim is to maximise the likelihood function subject to a constraint (instead of getting the maximum likelihood estimate) which is defined the circles around the point in coordinate (0, 0). Let the first contour curve be our desired restriction around the coordinate (0, 0), we can observe that it has a value of 0.15 which mean that restriction will be the following:

|  |  |
| --- | --- |
|  | (32) |

The restriction from equation (32) is equivalent to adding the penalisation term to the loss function in equation (31), meaning that the solution of the optimisation problem should be within the first circle around the point of coordinate (0, 0). More specifically, the point circled in red from the above graph.

This regularisation method is helpful to deal when multicollinearity occurs. Under these circumstances the estimates are unbiased, but their variances are large (far from the true values). By adding more bias to the estimated using ridge regression we reduce the standard errors making the estimates more stables and reliable, and therefore, reducing the variance of the model. Thus, the bias imposed by will shrink the collinear estimates towards zero but it will not be able to exclude them from the model unless get a very large value.

#### **2.1.3.2 Lasso Regression**

The Least Absolute Shrinkage and Selection Operator (lasso) regression. This method allows the exclusion of estimates from the model and works as a variable selection method by adding a penalty term that makes the potential solutions of the likelihood function to be restricted to a certain area in which the estimates are equal to zero when maximised the function. (Yang & Wen, 2018). This penalty can be observed as follows:

|  |  |
| --- | --- |
|  | (42) |

In this case the penalisation term is small than in ridge regression, note that ridge regression imposes the sum of squared betas, while lasso considers the absolute value of betas. Graphically speaking, we can observe in the following graph that equals zero when is between 0 and 0.28

Diagram, engineering drawing

Description automatically generated

The diamond shaped restriction is due to the absolute value from the penalisation term in the cost function. In this case the aim the lasso regression is to work as a variable selection method that reduces model complexity by decreasing the dimensionality of the model, therefore, the interpretation is simpler to interpret than in ridge regression which in contrast shrinks the estimates to zero, and even sometimes the parameter can change sign. In a summary, lasso also trades variance for bias but also has embedded a variable selection method.

#### **2.1.3.3 Elastic net**

The elastic netregularisation was introduced by (Zou & Hastie, 2005) which linearly combines the penalties terms used in the previous explained methods. This method combines both properties from the lasso and ridge regression. In this regularisation method we can find 2 tuning parameters which can be observed in the following formula:

|  |  |
| --- | --- |
|  | (43) |

It can be noted that is still a tuning parameter as in ridge and lasso, and in addition we find the parameter that weights the amount of penalty that comes from ridge and lasso. In this sense, when equals 0 ridge penalty is applied, when is lower than 0.5 the elastic net accounts more penalty weight from ridge, contrary to that, when is higher than 0.5 there is heavier lasso penalty, and lastly, when equals 1 only lasso penalisation takes place.

In the following graph we can observe that the contour curves for the restriction are not diamond-shaped nor circular-shaped, instead, is a combination of both. The restriction has smooth sides and sharped edges, allowing to remove estimates as in lasso, but depending of the tuning parameter it could only add bias to the estimates.

Diagram

Description automatically generated

The elastic net has an embedded grouping effect that allows correlated features to be discarded or included in the model. Also, (Zou & Hastie, 2005) proved that contrary to lasso, the elastic net method is useful when the number of variables is larger than the number of rows, this situation can be sometimes frequent in credit scoring, especially with behavioural data (Kaszyński et al., 2021).

It is important to note that (Zhou et al., 2014) demonstrated that the elastic net can be reduced to a support vector machine (SVM) which can be parallelised in a Graphics Processing Unit (GPU) for faster SVM solvers.

## **2.2 Non-Parametric models used in credit scoring**

Non-parametric models require a few or no assumptions concerning the relationship between dependent variable and explanatory variables, distribution of the variables or the errors include decision trees, random forest, extreme gradient boosting (XGBoosting), and neural network discussed below.

### **2.2.1 Decision Trees**

Advantages of classification trees

Disadvantages of classification trees

### **2.2.2 Random Forest**

### **2.2.3 Extreme Gradient Boosting**

### **2.2.4 Neural Networks**

## **2.3 Problem of multicollinearity**

## **2.4 Omitted variables problem**

## **2.5 Negative betas problem**

# **3. Important remarks in credit scoring modelling**

## **3.1 Obstacles in credit scoring**

### **3.1.1. Regulations**

#### **3.1.1.1 Basel I, II and III**

#### **3.1.1.2 Internal Ratings-Based Models**

#### **3.1.1.4 IFRS 9 Models**

#### **3.1.1.5 European Bank Authority guidelines**

### **3.1.1. Ethics**

## **3.2 Human oversight to the explainability of the model**

## **3.3 Fair awareness in credit scoring modelling**

# **4. Numerical example of credit scoring modelling**

## **4.1 Dataset definition: Home Credit Default Risk**

## **4.2 SEMMA Methodology definition**

## **4.3 Credit scoring modelling using regular variables**

### **4.3.1 Sample**

### **4.3.2 Explore**

### **4.3.3 Modify**

### **4.3.4 Model**

### **4.3.5 Assess**

## **4.4 Credit scoring modelling when adding sensitive variables**

### **4.4.1 Sample**

### **4.4.2 Explore**

### **4.4.3 Modify**

### **4.4.4 Model**

### **4.4.5 Assess**

## **4.5 Comparison of models**

# **5. Conclusions**

# **6. References**

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